Course Title: Engineering Mathematics (3) b Second Year (Computer & Automatic control Department)
Course Code: PME2111 Date:9 / 6 / 2013 (Second term) Allowed time: 3 hrs No. of Pages: (2)

Remarks: (Answer the following questions. Assume any missing data…)

Problem number 1(15 Marks)

(a) Find an analytic function whose real part is $u(x,y) = \sin x \cosh y$.

(b) Show that if $f(z) = u(r,\theta) + iv(r,\theta)$ is analytic, then $r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$.

(c) (Cauchy's Theorem) Prove that If f (z) is analytic in a simply-connected region D, then for every simple closed curve C in D, $\oint_C f(z)dz = 0$

Problem number 2(15 Marks)

(a)Evaluate

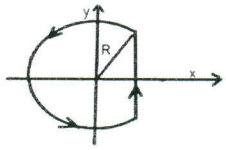
(i) $\oint_C \frac{z^3+1}{(z-1)(z-5)} dz$, $C: |z| = 3$. (ii) $\oint_C \frac{\sinh 3z}{(z-1)^4} dz$, $C: |z-1| = 3$.

(iii) $\oint_{|z|=2} z^2 \cosh \frac{2}{z-1}\, dz$

(b) Find Laurent's expansion of $f(z) = \frac{1}{z^2 - 3z + 2}$ on the regions

(i) $1 < |z| < 2$ (ii) $0 < |z-1| < 1$

(c) Using Bromwich contour



To find inverse Laplace transform of $F(s) = \frac{\cosh x\sqrt{s}}{s(s^2+1)}$, $0 < x < 1$

Problem number 3(15 Marks)

(a) Find sup(A),center of A, height of A and relative cardinality of ($\|A\|$)
where A is fuzzy set $A = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.2}{c} + \frac{0.8}{d}$

(b) Show that the set $A = \int \frac{\frac{1}{(1+x^2)}}{x}$ is convex

(c) Show that any membership μ_{AUB} satisfy S-norm axiom satisfy

$$max(\mu_A(x), \mu_B(x)) \le \mu_{AUB} \le \mu_{ds}(x)$$

where $\mu_{ds}(x) \begin{cases} \mu_A(x) & if \quad \mu_B(x) = 0 \\ \mu_B(x) & if \quad \mu_A(x) = 0 \\ 1 & otherwise \end{cases}$

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Problem number 4 (15 Marks)

(a) Show that Sugeno fuzzy complements class satisfies the complement Axioms. (hint:$C_\lambda(a) = \frac{1-a}{1+\lambda a}$, $0 < \lambda < \infty$ *and* $a = \mu_A(x)$)

(b) Find the fuzzy distance between elements of two fuzzy sets

$$A=\frac{0.1}{1}+\frac{0.3}{2}+\frac{0.7}{3} \quad \text{and} \quad B=\frac{0.5}{1}+\frac{0.2}{3}+\frac{0.6}{5}$$

(c) Let fuzzy set A be the set of people with an infectious disease and the crisp set B be the sets of people having been in contact with the infected people and C be the crisp set of people contact with B . The contact relations is given by R1 and R2

$$R_1 = \frac{0.8}{(a_1,b_1)}+\frac{0.2}{(a_2,b_2)}+\frac{0.3}{(a_3,b_1)}+\frac{0.7}{(a_4,b_2)}+\frac{0.4}{(a_4,b_3)} \quad , R_2 = \frac{0.1}{(b_1,c_1)}+\frac{0.2}{(b_2,c_2)}+\frac{0.5}{(b_3,c_1)}+\frac{0.9}{(b_3,c_3)}$$

A={(a1, 0.4), (a2,0.5), (a3, 0.9), (a4, 0.6)}, B = {b1, b2, b3} and C ={c1, c2, c3 }. Find the fuzzy sets $\widetilde{B}$ and $\widetilde{C}$.

Problem number 5 (25Mark)

(a) Evaluate the following integrals:

1) $\int_3^\infty e^{6x-x^2}\,dx$ 2) $\int_2^5 (x-2)\sqrt{5-x}\,dx$ 3) $\int_0^\infty \frac{dy}{1+y^4}$

(b) Find the series solution of the D.E. $(1+x^2)y'' + y = 0$

(c) Prove that: 1) $\frac{d}{dx}J_0(x) = -J_1(x)$ 2) $\int x^4 J_1(x)\,dx = x^4 J_2 - 2x^3 J_3 + c$

End

All best wishes

Prof. Dr. A. Abo Khadra , Dr .M. Shokry, Dr. Assem Elshenawy and the committee